Algebra • Patterns on the Multiplication Table

**Why Teach This**
Some students feel overwhelmed as they try to understand and remember multiplication facts. A multiplication table, which is mostly made up of number patterns, helps students see relationships among the facts. As students recognize and understand these patterns, they begin to see how the patterns become a learning tool. Looking for patterns and making generalizations also help students develop problem-solving skills.

**Professional Development Videos**

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**Learning Objective**
Identify and explain patterns on the multiplication table.

**Language Objective**
Students share with a partner how you can use properties to explain patterns on the multiplication table.

**Materials**
MathBoard

**Common Core State Standards**
3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

**Mathematical Practices**

MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP4 Model with mathematics.
MP5 Use appropriate tools strategically.
MP6 Attend to precision.
MP7 Look for and make use of structure.
MP8 Look for and express regularity in repeated reasoning.

**Standards Across the Grades**

**Before**
2.OA.A.3

**Grade 3**
3.OA.D.9 4.OA.C.5

**After**
4.OA.C.5

**Focus:**

Common Core State Standards
3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

Also 3.OA.B.5

**Mathematical Practices**

MP1 Make sense of problems and persevere in solving them.
MP3 Construct viable arguments and critique the reasoning of others.

**Coherence:**

Standards Across the Grades
Before  Grade 3  After
2.OA.A.3  3.OA.D.9  4.OA.C.5

**Rigor:**

Level 1: Understand Concepts..............Share and Show (☑ Checked Items)
Level 2: Procedural Skills and Fluency......On Your Own
Level 3: Applications........................Think Smarter and Go Deeper

**For more about how GO Math! fosters Coherence within the Content Standards and Mathematical Progressions for this chapter, see page 189J.**
Problem of the Day 4.7
There are 7 dogs staying at Bow Wow Beach Resort. Each dog eats 1 can of food a day. How many cans of food will the dogs eat in a week?

Vocabulary
• Interactive Student Edition
• Multimedia eGlossary

Essential Question
How can you use properties to explain patterns on the multiplication table?

Making Connections
Invite students to tell you what they know about patterns.

What is a pattern? something that repeats itself in an orderly way or according to a certain rule. What kind of patterns have you seen? Have you ever made a pattern yourself? Encourage students to think about different patterns in art, music, and mathematics.

Learning Activity
What is the problem the students are trying to solve? Connect the story to the problem.

• How many legs does each beetle have? 6 legs
• How could you find how many legs 2 beetles have in total? Multiply 2 × 6 or add 6 + 6.
• How could you find how many legs 3 beetles have in total? Multiply 3 × 6 or add 6 + 6 + 6.
• How does the total number of legs change from 1 to 2 to 3 beetles? Does it get smaller, larger, or stay the same? larger

Literacy and Mathematics
Choose one or more of the following activities.

• Write the word property on the board. Have students discuss the different meanings they know for this word. land someone owns; a rule or principle
• Show or ask students to draw a picture of a beetle. Have students write a list of six adjectives that could describe beetles.

Fluency Builder
Common Core Fluency Standard 3.OA.C.7

Mental Math Have students practice multiplication facts for 2 and 4. See how many they can answer in 30 seconds.

When all students are finished, discuss the strategies they used to complete the facts.

2 × 1 = 2 4 × 1 = 4
2 × 2 = 4 4 × 2 = 8
2 × 3 = 6 4 × 3 = 12
2 × 4 = 8 4 × 4 = 16
2 × 5 = 10 4 × 5 = 20
2 × 6 = 12 4 × 6 = 24
2 × 7 = 14 4 × 7 = 28
2 × 8 = 16 4 × 8 = 32
2 × 9 = 18 4 × 9 = 36
Unlock the Problem

MP5 Use appropriate tools strategically. Explain that students will use patterns and strategies to help them learn and remember multiplication facts.

Activity 1

Materials  MathBoard (see eTeacher Resources)

After they complete the products for the green squares, have them point to each product and say the corresponding number.

• Describe how the Commutative Property is shown on the table. Possible answer: the same product is shown twice in the green squares, but the factors are in a different order.

Math Talk

Use Math Talk to focus on students’ understanding of how to use patterns in a multiplication table to find products.

MP7 Look for and make use of structure. Have students fold the multiplication table in half along the diagonal from the upper left to the lower right. Next, have students shade the products on the diagonal fold. Have them write a multiplication sentence for some of the shaded products.

• What do you notice about the factors in each multiplication sentence? The factors are the same number.

ELL  Strategy: Model Language

Point to the multiplication table in the lesson.

• Explain and label the table with key vocabulary such as row, column, shade, product, and factor. Model the definition for each term.

• Have students work in teams to label their table.

• Have each member of the team choose a key term and explain its meaning to the group in their own words.

Reteach 4.7

Algebra • Patterns on the Multiplication Table

You can use a multiplication table to explore number patterns.

1 Activity 1 Possible answers are given.

Materials MathBoard

• Write the products for the green squares. What do you notice about the products?

I see products that are the same.

When I reach the product in the middle, they change order.

Write the multiplication sentences for the products on your MathBoard. What do you notice about the factors?

They have the same factors, but the factors are in a different order.

• Will this be true in the yellow squares? Explain using a property you know.

Yes, because of the Commutative Property of Multiplication.

Write the products for the yellow squares.

• Complete the columns for 1, 5, and 6. Look across each row and compare the products. What do you notice?

The products for 6 are the sums of the products for 1 and 5.

What property does this show?

Distributive Property

Enrich 4.7

Pattern Products

Follow the directions for the multiplication table.

a. Shade all of the products in the row and column for 2.

b. Circle all of the products in the row and column for 4.

c. Describe the pattern in the products that are shaded or circled.

Possible answer: all the products are even.

The products of 4 are the products of 2 doubled.

Math Talk: Possible answer: I can add the products for the columns for 1 and 6 to find the products for the column for 7.

Unlock the Problem

Pattern Products

Follow the directions for the multiplication table.

a. Shade all of the products in the row and column for 2.

b. Circle all of the products in the row and column for 4.

c. Describe the pattern in the shaded products.

Possible answer: the numbers that are shaded and circled are products of 2 and 4.

Stretch Your Thinking: Shade the row and column for 8.

Check whether some of the shaded products have the same number of cells.

Possible answer: the products of 8 are the products of 4 doubled; the products of 8 are 4 times as great as the products of 4; all products are even.
Activity 2

Before students begin the activity, ask them to review the meaning of even and odd numbers. As students work on the activity, continue to focus on how patterns can help them remember multiplication facts.

- How can you use patterns and properties to help you multiply by 0 and 1? The product of 0 and a number is always 0. The product of 1 and a number is always that number.

After discussing any patterns they see, ask the following questions:
- Are all the products for 6 even or odd? even
- Is the product even or odd? 30 even

Write the following on the board:
\[(3 + 3) \times 5 = (3 \times 5) + (3 \times 5)\]
- What is the product? 30 Is the product even or odd? even

- What property did we use to show this pattern? Distributive Property
- How does this property explain the pattern? Possible answer: I know that an odd number added to another odd number will always be an even number. So, any odd number multiplied by 6 will be even.

Have students use the Distributive Property to show that any even number multiplied by 6 will also have a product that is even. Students should provide an example.

3 EXPLAIN

Share and Show

The first problem connects to the learning model. Have students use the MathBoard to explain their thinking.

Math Talk

Use Math Talk to focus on students’ understanding of patterns in the multiplication table.

COMMON ERRORS

Error Students confuse odd and even numbers.

Example Is the number 40 odd or even? Student answer: odd

Springboard to Learning Write the numbers 0, 2, 4, 6, 8 on a strip of paper. Remind students that if a number ends with one of these numbers, it is even.
Use the checked exercises for **Quick Check**.

**Quick Check**

If a student misses the checked exercises

Then **Differentiate Instruction** with

- Reteach 4.7
- Personal Math Trainer 3.OA.D.9
- RtI Tier 1 Activity (online)

**On Your Own**

If students complete the checked exercises correctly, they may continue with the **On Your Own** section.

**GO DEEPER**

**MP7** Look for and make use of structure. Exercise 12 requires students to analyze a pattern and find an error. Analyzing information helps students check their understanding of multiplication patterns.

**4 ELABORATE**

**Problem Solving • Applications**

Have students read Exercise 13. Ask them to explain how they will solve the problem.

**THINK SMARTER**

**Math on the Spot**

**Video Tutor**

Use this video to help students model and solve this type of Think Smarter problem.


**Personal Math Trainer**

Be sure to assign this problem to students in the Personal Math Trainer. It features an animation to help them model and answer the problem. To be able to complete the table, students must know that the product of two odd numbers is odd, the product of two even numbers is even, and the product of one odd and one even number is even. Students also might recognize the pattern alternating between odd and even across the table.

**Problem Solving • Applications**

Complete the table. Then describe a pattern you see in the products.

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
</tr>
</tbody>
</table>

Possible answers: the ones digit is always zero; the products are even.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
<td>15</td>
<td>25</td>
<td>35</td>
<td>45</td>
</tr>
</tbody>
</table>

Possible answers: the ones digit is always 5; the products are odd.

**SMARTER** Explain how patterns of the ones digits in the products relate to the factors in Exercises 13 and 14.

Possible explanation: when an even factor is multiplied by 5, the ones digit is always zero; when an odd factor is multiplied by 5, the ones digit is always 5.

**SMARTER** Helene selected an odd number to multiply by the factors in this table. Write even or odd to describe each product.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>odd number</td>
<td>odd</td>
<td>even</td>
<td>odd</td>
<td>even</td>
<td>odd</td>
</tr>
</tbody>
</table>

**Final Thoughts**

Review the reinforcement and extension activities. Ask students to describe their strategies for solving Exercises 13 and 14.

In your classroom:

- **Reteach** for students who need additional support.
- **Personal Math Trainer** for personalized practice.
- **Math on the Spot** videos for additional explanations.

**Quick Check**

Use the multiplication table. Describe a pattern you see. Possible patterns are given.

6. in the column for 10
   - The ones digit is always 0; each number is 10 more than the number above it.

7. in the column for 8
   - The ones digit repeats—0, 8, 6, 4, 2;
   - the products are all even.

**On Your Own**

Is the product even or odd? Write **even** or **odd**.

8. \(4 \times 8\) **even**

9. \(5 \times 5\) **odd**

10. \(7 \times 4\) **even**

11. \(2 \times 9\) **even**

12. Use the multiplication table. Rewrite the correct pattern.

   6, 12, 18, 24, 30, 36

   6, 12, 18, 22, 30, 36

   6, 12, 18, 24, 30, 36

**Problem Solving • Applications**

Chapter 4 • Lesson 7  231
Sense or Nonsense?

17. **Make Arguments** Whose statement makes sense? Whose statement is nonsense? Explain your reasoning.

The product of an odd number and an even number is even.

Gunter’s Work

<table>
<thead>
<tr>
<th>odd</th>
<th>even</th>
<th>even</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
<td>12</td>
</tr>
</tbody>
</table>

I can circle 2 equal groups of 6 with no tiles left over. So, the product is even.

Giselle’s Work

<table>
<thead>
<tr>
<th>even</th>
<th>even</th>
<th>even</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

I can circle 6 pairs with no tiles left over. So, the product is even.

Possible explanation: both statements make sense. They both show how the product can be an even number.

18. **Go Deeper** Write a statement about the product of two odd numbers. Give an example to show why your statement is true.

Possible answer: the product of two odd numbers is an odd number. I can circle 2 equal groups of 7 with 1 tile left over. So, the product is odd.

Math Journal

**WRITE**

Draw a picture that shows an example of a product of two even numbers. Write the matching multiplication sentence.

**DIFFERENTIATED INSTRUCTION INDEPENDENT ACTIVITIES**

**Grab-o-Go!** Differentiated Centers Kit

**Activities**

*Diamond Derby*

Students complete purple Activity Card 15 by practicing multiplication facts through 10 by 10.

**Literature**

*Here’s What I Do*

Students read about using multiplication tables to win a computer game about multiplication.

**Games**

*Multiplication Bingo*

Students practice multiplication facts through 10.

**Essential Question**

**Go Deeper**

For Exercise 18, be sure students write a statement and give an example either as a multiplication sentence and/or a drawing that supports their statement.

For Exercise 17, have students compare Gunter’s and Giselle’s work. Elicit from students that Gunter’s work shows 2 equal-sized groups and Giselle’s work shows groups of 2.

As students analyze Gunter’s work and Giselle’s work, guide them to see that when an even number is multiplied by a number, the product can be grouped into two equal addends.

- **Look at Gunter’s work and his drawing.** How did Gunter show the product of 12 as two equal addends? He showed 12 as 6 + 6. Both addends are 6, so they are equal addends.

Encourage students to use other multiplication sentences to reinforce this concept. Have them use drawings or counters to illustrate their findings.

**Go Deeper**

For Exercise 18, be sure students write a statement and give an example either as a multiplication sentence and/or a drawing that supports their statement.

**Reflect** Have students share with a partner to answer the Essential Question.

How can you use properties to explain patterns on the multiplication table? I can use the Commutative Property to find patterns that show the same factors, but in a different order. I can also use the Associative Property and Distributive Property to find patterns for products that are even or odd.

**Math Journal**

Draw a picture that shows an example of a product of two even numbers. Write the matching multiplication sentence.
Practice and Homework

Use the Practice and Homework pages to provide students with more practice of the concepts and skills presented in this lesson. Students master their understanding as they complete practice items and then challenge their critical thinking skills with Problem Solving. Use the Write Math section to determine student’s understanding of content for this lesson. Encourage students to use their Math Journals to record their answers.

Patterns on the Multiplication Table

Is the product even or odd? Write even or odd.

1. \(2 \times 7 = \) **even**  
   Think: Products with 2 as a factor are even.

2. \(4 \times 6 = \) **even**

3. \(8 \times 3 = \) **even**

Use the multiplication table. Describe a pattern you see. Possible patterns are given.

4. in the column for 5
   The ones digits repeat—0, 5; each number is 5 more than the number above it.
   
<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
<td>0</td>
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<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

5. in the row for \(10\)
   Add 10; all the products are even; the ones digit is always 0.
   
<table>
<thead>
<tr>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
</tr>
</tbody>
</table>

6. in the rows for 3 and 6
   The products of 6 are the products of 3 doubled.
   
<table>
<thead>
<tr>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
<th>18</th>
<th>21</th>
<th>24</th>
<th>27</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>36</td>
<td>42</td>
<td>48</td>
<td>54</td>
<td>60</td>
</tr>
</tbody>
</table>

Problem Solving

7. Carl shades a row in the multiplication table. The products in the row are all even. The ones digits in the products repeat 0, 4, 2, 6. What row does Carl shade?

   - the row for 4

8. Jenna says that no row or column contains products with only odd numbers. Do you agree? Explain.
   
   Yes. Possible explanation: either the products are all even, or there is an even and odd number pattern.

9. **WRITE**  
   Draw a picture that shows an example of a product of two even numbers. Write the matching multiplication sentence.
   
   Check students’ work.

Mathematical Practices in Your Classroom

**CCSS.Math.Practice.MP3** Construct viable arguments and critique the reasoning of others.

Mathematically proficient students will be able to analyze situations and justify their conclusions. In this lesson, students can use the multiplication chart to justify their conclusions and explain them to others. Other students can respond to the reasoning of their classmates and determine if their reasoning is flawed.

Ask questions such as the following to help students critique the reasoning of others:

**Teacher:** What conclusions did you draw as you worked on Exercise 4?

**Tony:** I made the conclusion that each number is 5 more than the number above it.

**Teacher:** Do you all agree with Tony’s conclusion? Why or why not?
Lesson Check (3.OA.D.9)

1. Is the product of $4 \times 9$ even or odd?
   
   Possible answers: even

2. Describe a pattern you see.
   
   10, 15, 20, 25, 30

   Possible answers: add 5, even and then odd

Spiral Review (3.OA.A.3, 3.OA.B.5, 3.NBT.A.2, 3.MD.B.3)

3. Lexi has 2 cans of tennis balls. There are 3 tennis balls in each can. She buys 2 more cans. How many tennis balls does she now have?

   12 tennis balls

4. Use the picture graph.

   How many students have green eyes?

   16 students

5. Sasha bought 3 boxes of pencils. If each box has 6 pencils, how many pencils did Sasha buy?

   18 pencils

6. Find the sum.

   $219 + 763 = 982$